B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

Subject : Physics

Course : DSE-2(OR)

(Classical Dynamics)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any ten* questions from the following: 2×10=20

(a) Prove that the kinetic energy of a charged particle moving in a magnetic field remains constant.

(b) Define generalised coordinates. Mention the generalised coordinates of a particle constrained to move on a cylindrical surface.

(c) A particle of mass m slides under gravity without friction along a cycloid described by equations $x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$, where a is constant. Find the Lagrangian of the particle.

(d) Find the position of stable equilibrium of a particle in the potential $V(x) = \frac{kx^2}{2} - \frac{\lambda x^3}{3}$, where $k, \lambda > 0$.

(e) The Lagrangian of a particle $L = \frac{m(\dot{q}_1^2 + \dot{q}_2^2)}{2} - \frac{\lambda}{2}(q_1^2 + q_2^2)$. Find the Hamiltonian of a system.

(f) Show that the Hamiltonian of a system remains constant if the Lagrangian of the system does not depend upon time explicitly.

(g) The equations of motion of an oscillatory system is

$$\ddot{x} + \omega_0^2 x - \alpha y = 0$$

$$\ddot{y} + \omega_0^2 y - \alpha x = 0.$$

Find the normal coordinates and normal frequencies of the system.

(h) If the half-life of an elementary particle moving with speed 0.9c in the laboratory frame is 5×10^{-8}s, then what is the proper half-life of the particle?

(i) The kinetic energy of a particle of rest mass m_0 is equal to its rest mass energy. Find its velocity.

(j) Explain the simultaneity of two events in special theory of relativity.

(k) Define world point and world line in special theory of relativity.

(l) Define space-time interval. What will be the space-time interval at the surface of light-cone?

(m) Show that the Lorentz transformation is the transformation from orthogonal to non-orthogonal coordinate system (x, ct).

(n) The velocity components of a fluid (incompressible) flow are

$$u = x^2 + z^2, v = y^2 + z^2, w = -2z(x + y).$$

Show that the fluid flow is possible.

(o) Two capillary tubes of radii R_1, R_2 and lengths L_1, L_2 respectively are connected in series. A fluid flows through the tube due to pressure difference P at two free ends. Using Poiseuille's equation find the expression of pressure in the fluid at the junction of two tubes.

2. Answer *any four* questions from the following: 5×4=20

(a) A charge particle initially moving with velocity $\vec{u_0}$ enters a uniform electric field $\vec{E}$ in transverse direction. Show that the path of the particle is a parabola.
The electric field is produced by applying potential difference between two parallel conducting plates of length l. A fluorescent screen is placed at a distance D from the middle of the plates. Obtain an expression of the shift of the spot on the screen due to applied electric field. 3+2

(b) (i) What is cyclic coordinate? Show that the conjugate momentum corresponding to a cyclic coordinate is conserved.

(ii) If the Lagrangian of a system does not depend upon time explicitly, show that the total energy of the system remains constant using Lagrangian formalism. (1+2)+2

(c) (i) The Hamiltonian of a system is $H = \frac{\alpha p^2 q^4}{2} + \frac{\beta}{q^2}$, where α and β are constants, and q and p are the generalized coordinate and momentum. Find the Lagrangian of the system.

(ii) The Hamiltonian of a system with generalised coordinate and momentum (q, p) is $H = p^2 q^2$. Show that $p = Be^{-2At}$, where A and B are constants. 2+3

(d) (i) Four-momentum is defined as $P_\mu = (\vec{p}, iE/c)$, where $\vec{p}$ is three-momentum vector, E is the energy and c is the speed of light in vacuum. Show that the four-force can be expressed as $F_\mu = \left(\gamma\vec{f}, \frac{i\gamma}{c}\vec{u}.\vec{f}\right)$, where $\vec{f}$ is three-force vector and $\vec{u}$ is the velocity of particle.

(ii) Show that four-velocity and four-force are orthogonal to each other. 3+2

(e) (i) Explain the length-contraction in four-dimensional space $(\vec{r}, ict)$.

(ii) A cube of side l_0 moves with velocity u along one of the edges with respect to a reference frame. Find the volume of the cube in that frame. 3+2

(f) (i) Using the equation of continuity show that the velocity potential satisfies Laplace equation for incompressible and irrotational fluid.

(ii) What is streamline of fluid flow? The velocity field is $\vec{v} = (1 + At)\hat{\imath} + x\hat{\jmath}$. Find the equation of streamline at $t = t_0$ passing through point (x_0, y_0). 2+(1+2)

3. Answer *any two* questions from the following: 10×2=20

(a) (i) State the principle of Hamilton's least action. Hence, derive Euler-Lagrange's equation of motion.

(ii) A particle of mass m is constrained to move on a circle under gravity. The circle is placed in a vertical plane and placed on the ground. Mention the generalised coordinates. Find the Lagrange's equation of motion. (2+4)+(1+3)

(b) Consider a linear tri-atomic molecule CO_2 in which Oxygen atoms are connected to the Carbon atom by springs with spring constant k. Write down the Lagrangian of the system. Hence, obtain the secular equation. Determine the normal frequencies and obtain the relation among amplitudes of the atoms at normal frequencies. (1+3)+(2+4)

(c) What is central force field? Write down the Lagrangian of a particle moving under central force field. Hence, derive the Hamiltonian and Hamilton's equations of motion of the particle. Using Hamilton's equations show that the angular momentum and total energy of the particle remain conserved. 1+(1+2+2)+(1+3)

(d) (i) Write down the Lorentz transformation of four-momentum. Hence, explain the Doppler effect and relativistic aberration of light.

(ii) Show that $A_\mu B_\mu$ is invariant under Lorentz transformation. (2+3+2)+3

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